MATRIKS INVERS

$$A^{-1} = \frac{1}{|A|} \cdot \mathrm{adj}(A)$$

$$\lim_{x \to 0} \frac{1}{x} =$$

SOFTWARE MATEMATIKA :

1. MATHEMATICA $\rightarrow$ WOLFRAM RESEARCH
2. MAPLE
3. MATLAB

PERKALIAN SKALAR DGN SUATU MATRIKS

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 6 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 & 3 \\ 12 & 15 & 18 \end{bmatrix}$$

$$A \cdot B = B \cdot A = I$$

$$B = A^{-1}, \quad A = B^{-1}$$

$$\boxed{A \cdot A^{-1} = I}, \quad \boxed{B \cdot B^{-1} = I}$$

(1) A non singular $\rightarrow |A| \neq 0$

B ——— " ——— $\rightarrow |B| \neq 0$

$|A \cdot B| \neq 0 \rightarrow A \cdot B$ MATRIKS NON Singular

(2) $(A$

(3) A

(4) $\boxed{A}$

BUKT

A

$= I$

$^{-1} = I$

0

0

ATRIKS

ON Singular

② $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$

③ $A^3 = A \cdot A \cdot A$

④ $\boxed{A^0 = I}$?

BUKTI :

$\underbrace{A \cdot A^{-1}} = I$

$A^0 = I$

5 $A^{-3} = (A^{-1})^{3} = A^{-1} \cdot A^{-1} \cdot A^{-1}$

6 $(A^{-1})^{-1} = A$

7 $(3A)^{-1} = 3^{-1} \cdot A^{-1}$

$= \frac{1}{3} A^{-1}$

8 $A^{3} \cdot A^{2} = A^{5}$

9 $(A^{3})^{2} = A^{6}$

A^{-1}

MENCARI MATRIKS INVERS

1. $A \cdot A^{-1} = I$

2. $(A \mid I) \xrightarrow{\text{OBE}} (I \mid A^{-1})$

3. $A^{-1} = \dfrac{1}{|A|} \cdot \text{adj}(A)$

$$A_{2\times 2} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \longrightarrow A^{-1} = ? \quad , \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\textcircled{I} \quad \boxed{A \cdot A^{-1} = I} \qquad \text{MIS: } A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{array}{rl} a+2c = 1 & \times 2 \\ 3a+4c = 0 & \times 1 \end{array} \qquad \begin{array}{rl} b+2d = 0 & \times 3 \\ 3b+4d = 1 & \times 1 \end{array}$$

$$\begin{array}{r} 2a+\cancel{4c} = 2 \\ 3a+\cancel{4c} = 0 \end{array} \ominus \qquad \begin{array}{r} \cancel{3b}+6d = 0 \\ \cancel{3b}+4d = 1 \end{array} \ominus$$

$$-a = 2 \qquad 2d = -1$$

$$\boxed{a = -2} \qquad \boxed{d = -\frac{1}{2}}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a + 2c = 1$$
$$-2 + 2c = 1$$
$$2c = 1 + 2$$
$$2c = 3$$
$$c = \frac{3}{2} = 1\tfrac{1}{2}$$

$$b + 2d = 0$$
$$b = -2d$$
$$b = -2\left(-\tfrac{1}{2}\right)$$
$$b = 1$$

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 1 \\ 1\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$A_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \longrightarrow A^{-1} = \frac{1}{a_{11}.a_{22} - a_{12}.a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

MIS : $A^{-1} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$$\boxed{A \cdot A^{-1} = I_2}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_{11}.p + a_{12}.r & a_{11}.q + a_{12}.s \\ a_{21}.p + a_{22}.r & a_{21}.q + a_{22}.s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a_{11} \cdot p + a_{12} \cdot r = 1 \quad \Big| \times a_{22}$$
$$a_{21} \cdot p + a_{22} \cdot r = 0 \quad \Big| \times a_{12}$$

$$a_{11} \cdot a_{22} \cdot p + \cancel{a_{12} \cdot a_{22} \cdot r} = a_{22}$$
$$a_{21} \cdot a_{12} \cdot p + \cancel{a_{12} \cdot a_{22} \cdot r} = 0 \quad \ominus$$

$$(a_{11} \cdot a_{22} - a_{21} \cdot a_{12}) \cdot p = a_{22}$$

$$\boxed{p = \frac{a_{22}}{a_{11} \cdot a_{22} - a_{12} \cdot a_{21}}}$$

$$a_{11} \cdot p + a_{12} \cdot r = 1 \quad \Big| \times a_{21}$$
$$a_{21} \cdot p + a_{22} \cdot r = 0 \quad \Big| \times a_{11}$$

$\times a_{22}$

$\times a_{12}$

$\cdot r = a_{22}$

$r = 0 \ominus$

$= a_{22}$

a_{21}

a_{11}

$$\cancel{a_{11} \cdot a_{21} \cdot p} + a_{12} \cdot a_{21} \cdot r = a_{21}$$
$$\cancel{a_{11} \cdot a_{21} \cdot p} + a_{11} \cdot a_{22} \cdot r = 0 \ominus$$

$$(a_{12} \cdot a_{21} - a_{11} \cdot a_{22})\, r = a_{21}$$

$$-(a_{11} \cdot a_{22} - a_{12}\, a_{21})\, r = a_{21} \quad \times(-1)$$

$$(a_{11} \cdot a_{22} - a_{12} \cdot a_{21})\, r = -a_{21}$$

$$\boxed{r = \frac{-a_{21}}{a_{11} \cdot a_{22} - a_{12} \cdot a_{21}}}$$

$$a_{11} \cdot q + a_{12} \cdot s = 0 \quad \times a_{22}$$
$$a_{21} \cdot q + a_{22} \cdot s = 1 \quad \times a_{12}$$

$$a_{11} \cdot a_{22} \cdot q + \cancel{a_{12} \cdot a_{22} \cdot s} = 0$$
$$a_{12} \cdot a_{21} \cdot q + \cancel{a_{12} \cdot a_{22} \cdot s} = a_{12} \ominus$$

$$(a_{11} \cdot a_{22} - a_{12} \cdot a_{21}) \cdot q = -a_{12}$$

$$\boxed{q = \frac{-a_{12}}{a_{11} \cdot a_{22} - a_{12} \cdot a_{21}}}$$

$a_{11} \cdot q + a_{12}$

$a_{21} \cdot q + a_{2}$

$\cancel{a_{11} \cdot a_{21} \cdot q} +$

$\cancel{a_{11} \cdot a_{21} \cdot q} +$

$(a_{12} \cdot a_{21}$

$\ominus (a_{11} \cdot a$

$s =$

$r = a_{21}$

$r = 0 \ominus$

$= a_{21}$

$= a_{21}$

$\times (-1)$

$-a_{21}$

a_{22}

12

$$\begin{array}{l|l} a_{11} \cdot q + a_{12} \cdot s = 0 & \times a_{21} \\ a_{21} \cdot q + a_{22} \cdot s = 1 & \times a_{11} \end{array}$$

$$a_{11} \cdot a_{21} \cdot q + a_{12} \cdot a_{21} \cdot s = 0$$

$$a_{11} \cdot a_{21} \cdot q + a_{11} \cdot a_{22} \cdot s = a_{11} \quad \ominus$$

$$(a_{12} \cdot a_{21} - a_{11} \cdot a_{22}) \cdot s = -a_{11}$$

$$\ominus(a_{11} \cdot a_{22} - a_{12} \cdot a_{21}) \cdot s = \ominus a_{11}$$

$$\boxed{s = \frac{a_{11}}{a_{11} \cdot a_{22} - a_{12} \cdot a_{21}}}$$

$$A^{-1} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{a_{22}}{a_{11}.a_{22}-a_{12}.a_{21}} & \dfrac{-a_{12}}{a_{11}.a_{22}-a_{12}.a_{21}} \\[2ex] \dfrac{-a_{21}}{a_{11}.a_{22}-a_{12}.a_{21}} & \dfrac{a_{11}}{a_{11}.a_{22}-a_{12}.a_{21}} \end{bmatrix}$$

$$\frac{1}{a_{11}.a_{22}-a_{12}.a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, \quad B^t = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

① $(A^t)^t = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}^t = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = A$

② $A + B = \begin{bmatrix} 5 & 4 \\ 5 & 6 \end{bmatrix} \longrightarrow (A+B)^t = \begin{bmatrix} 5 & 5 \\ 4 & 6 \end{bmatrix}$

$$A^t + B^t = \begin{bmatrix} 5 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\longrightarrow (A+B)^t = A^t + B^t$$

③ $5A = 5\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 5 & 20 \end{bmatrix}$

$(5A)^t = \begin{bmatrix} 10 & 5 \\ 15 & 20 \end{bmatrix}$

$5 \cdot A^t = 5\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} = \begin{bmatrix} 10 & 5 \\ 15 & 20 \end{bmatrix}$

$\boxed{(5A)^t = 5A^t}$

$\begin{bmatrix} 10 & 15 \\ 5 & 20 \end{bmatrix}$

$\parallel$

$\begin{bmatrix} 10 & 5 \\ 15 & 20 \end{bmatrix}$

(4) $A \cdot B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 6+12 & 2+6 \\ 3+16 & 1+8 \end{bmatrix} = \begin{bmatrix} 18 & 8 \\ 19 & 9 \end{bmatrix}$

$$(A \cdot B)^{-1} = \frac{1}{(18)(9)-(8 \times 19)} \begin{bmatrix} 9 & -8 \\ -19 & 18 \end{bmatrix}$$

$$= \frac{1}{162-152} \begin{bmatrix} 9 & -8 \\ -19 & 18 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 9 & -8 \\ -19 & 18 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \rightarrow A^{-1} = \frac{1}{8-3} \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \rightarrow B^{-1} = \frac{1}{6-4} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$B^{-1} \cdot A^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} \cdot \frac{1}{5} \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 8+1 & -6-2 \\ -16-3 & 12+6 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 9 & -8 \\ -19 & 18 \end{bmatrix}$$